

Year of Approval by Department: 1977

Mathematics 4206[12.1]: Advanced Calculus II – Functions of Several Variables
4 hours lecture; 4 credits

Prerequisites: Mathematics **2101 [10.1]** and **4201 [11.1]** and one other advanced Mathematics Department course

Frequency of Offering: Once a year

Projected Enrollment: 25 students

Topics

- I. \mathbb{R}^n – open and closed sets, convergence of sequences of points in \mathbb{R}^n , continuous functions defined on subsets of \mathbb{R}^n , vector-valued functions, curves.
- II. Differentiation – partial derivatives, differentials, differentiable functions, sufficient conditions for functions to have differential, chain rule (with proof), the mean-value theorem and Taylor's theorem, computational of extrema, implicit function theorem and inverse function theorem (with proofs), Jacobians.
- III. Integration – Riemann-Darboux integrals of bounded functions, properties of integrals, statement and proof of necessary and sufficient condition for a function to be Riemann integrable in terms of upper and lower sums, and applications of condition to prove the existence of the integral of continuous functions and some discontinuous functions. The fundamental theorem of Calculus, mean-value theorem for integrals. Improper integrals.
Multiple integrals – sets of content 0 in \mathbb{R}^n . Definition of n-dimensional multiple Riemann integrals over a rectangle, and existence of integral for a function whose set of discontinuities has content 0. Iterated integrals. Change of variables theorem in multiple integrals (with proofs for suitably defined regions whenever possible). Uniform convergence, improper multiple integrals, integrals depending on a parameter.
- IV. Line and surface integrals – arc length, line and surface integrals, exact differentials and independence of path for line integrals, Green's theorem, Stokes' theorem, divergence theorem.