Math 3111: Graph Theory and Applications

Instructor: Dr. Sandra Kingan
Office: 2317bN (Old Ingersoll)
URL: sandrakingan.com

Course Description: A rigorous study of graph theory and algorithms. Topics include trees, connectivity, Eulerian and Hamiltonian graphs, planar graphs, graph colorings, network flows, matching theory, graph minors, and applications to large graphs.

Course Objectives:
- To gain a thorough understanding of the fundamental concepts of graph theory including isomorphism, trees, connectivity, higher connectivity, coloring, flows, matching theory, and planarity.
- To understand graph algorithms such as greedy algorithms, shortest path algorithm, and algorithms for finding maximum matchings.
- To understand and appreciate the wide variety of applications to subjects within and outside mathematics such as data science, computer science, biology, psychology, chemistry, and social and political sciences

Prerequisite: Math 2101: Linear Algebra

Textbook: *Graphs and Networks* by S. R. Kingan, published by Wiley.

Course Topics
- Introduction and Course Framing (From Konigsberg to Connectomes – theory with algorithms and modern applications to network science)
- Graph Isomorphism (labeled and unlabeled)
- Graph Constructions and Substructures (including minors)
- Trees (basic results as well as proof of Cayley's tree Counting Theorem, Kirchoff's Matrix Tree Theorem)
- Distance in graphs (eccentricity, diameter, radius, mean distance)
- Degree Sequences (proof of Havel-Hakimi Theorem, Erdős-Gallai Theorem)
- Centrality Measures (path-based measures and walk-based measures such as eigenvector centrality and Google's PageRank)
- Similarity Measures (Jaccard similarity, recommendation systems, clustering coefficient) (optional topic)
- Small-World Graphs and Scale-Free Graphs (optional topic)
- Graph Algorithms (Depth-First search, Breadth-First Search, Greedy Algorithms for minimum weight spanning tree (Prim and Kruskal), proof that the Greedy Algorithm works, Dijkstra's Shortest Path Algorithm
- Eulerian Circuits (proof of Euler's Theorem and Veblen's cycle decomposition theorem, Eulerizing a graph)
- Hamiltonian Cycles (proof of Ore's Theorem)
- Coloring (bounds on vertex and edge coloring, greedy coloring algorithm, proof of Brook's vertex coloring theorem, Vizing's edge coloring theorem, snarks)
- Higher Connectivity (vertex and edge connectivity, ear decomposition of 2-connected graphs, Menger's theorem, Whitney's k-connected theorem).
- Properties of Planar Graphs
- Polyhedra and proof of Euclid's Theorem on 5 regular Platonic solids
- Four Color Theorem (statement of 4-color theorem and proof of 5-color theorem)
- Kuratowski-Wagner's excluded minor theorem for planar graphs (optional)
- Non-planar graphs (Crossing number, Thickness, genus) (optional)
- Flows in Networks (proof Max-flow min cut theorem)
- Stable Sets, Matchings, Coverings (proof of Konig's theorem)
- Min-Max Theorems and Edmonds' Blossom Algorithm (optional)

Evaluation: Your understanding of the course material will be evaluated through collaborative homework (20%), two tests (50%) and a final exam (30%). You may not give nor receive any help during the two tests and the final exam. A make-up for an exam is available only in case of a medical emergency and will require documentation in the form of a doctor's note.

Disability Support: The Center for Student Disability Services (CSDS) addresses all matters related to disabilities. In order to receive disability-related academic accommodations students must first be registered with CSDS. If you have already registered with CSDS, please email CDSC to ensure the accommodation email is sent to your professor.

Student Bereavement Policy:
http://www.brooklyn.cuny.edu/web/about/initiatives/policies/bereavement.php

Religious Beliefs: The state law regarding non-attendance because of religious beliefs may be found on p. 66 in the *Undergraduate Bulletin* or p. 40 in the *Graduate Bulletin*.

Academic Integrity: Please see CUNY's Academic Integrity Policy.

Fall 2024 Important dates:

- Thu Aug 29: First Class
- **Tue Oct 1: Test 1**
- Thu Oct 3: Holiday (Rosh Hashanah)
- Tue Oct 15: No class for this course (Tue with a Mon schedule)
- **Thu Nov 7: Test 2**
- Thu Nov 28: Holiday (Thanksgiving)
- Thu Dec 12: Last Class
- Dec 16 – 20: Final Exams